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SCIENCE, volume 55, June 23, 1922: "Are scientists encouraging popular ignorance?" by E. C. Bingham, 664–667 [Considers the objections to the metric system in the report of the National Industrial Conference Board.]—Volume 56, July 14: "Methods of German publishers" by R. C. Archibald, 45 ["In Germany the chief publishers of mathematical books and periodicals are Springer, Teubner, and Vereinigung wissenschaftlicher Verleger (a combination of the firms: Göschen, Guttentag, Reimer, Trübner and Veit). They have decided that for their mathematical publications of 1922 Americans shall, in general, be required to pay at least as much as $2.40 per 100 marks of the price for Germany. Of *Jahrbuch über die Fortschritte der Mathematik*, volume 45 . . . part 1 . . . and part 2 . . . are sold in Germany for 73 and 190 marks respectively. The corresponding prices for America are $4.65 and $9.00! Such extortion ought appreciably to hasten the appearance of an American abstract journal, the establishment of which has been already approved by the National Research Council. But again, *Journal für die reine und angewandte Mathematik* (Crelle), volume 151 (1920–21), is sold in Germany for 96 marks; the price to America is $6.00! . . . The above facts, obtained from the publishers themselves on May 26 and May 31, 1922, will probably suggest to mathematicians the immediate cancellation of all contemplated orders for the publications of Vereinigung wissenschaftlicher Verleger—at least."]

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL, OTTO DUNKEL, AND H. P. MANNING.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N. B. Problems containing results believed to be new, or extensions of old results, are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist the members of the Association with their difficulties in the solution of such problems.]

2981. Proposed by H. P. MANNING, Providence, R. I.

Find the envelope of the circle on which two diametrically opposite points divide in a given ratio the focal radii of a variable point on an ellipse or hyperbola.

2982. Proposed by H. P. MANNING, Providence, R. I.

Solve the following problem without using analytic geometry: A triangular yard has a post at each corner. Given the lengths of the sides of the yard and of the posts, find the length and the position of the foot of a ladder that will just reach the top of each post without changing this position.

2983. Proposed by R. S. UNDERWOOD, Alabama Polytechnic Institute.

Prove that the following terminating series have the sums indicated, where by 0! is meant 1:

$$(1)\quad \frac{1}{n!\,1!}+\frac{1}{(n-2)!\,3!}+\frac{1}{(n-4)!\,5!}+\cdots=\frac{2^n}{(n+1)!},$$

$$(2)\quad \frac{1}{(n+1)!\,0!}+\frac{1}{(n-1)!\,2!}+\frac{1}{(n-3)!\,4!}+\cdots=\frac{2^n}{(n+1)!}.$$

(These coincide for n even but are distinct for n odd),

$$(3)\quad \frac{1}{n!\,1!}-\frac{1}{(n-2)!\,3!}+\frac{1}{(n-4)!\,5!}-\cdots=\frac{\alpha(-4)^{[n/4]}}{(n+1)!}$$

where $[n/4]$ denotes the greatest integer not exceeding $n/4$, and where $\alpha = 1, 2, 2, 0$ according as n is congruent to 0, 1, 2, 3 (mod 4).

Obtain a similar form for the series

$$(4)\quad \frac{1}{(n+1)!\,0!}-\frac{1}{(n-1)!\,2!}+\frac{1}{(n-3)!\,4!}-\cdots.$$

Obtain also the sums of the following terminating series and of the four series obtained from them by changing the signs of the alternate terms:

$$(5)\quad \frac{1}{n!\,1!} + \frac{1}{(n-4)!\,5!} + \frac{1}{(n-8)!\,9!} + \cdots,$$

$$(6)\quad \frac{1}{(n+1)!\,0!} + \frac{1}{(n-3)!\,4!} + \frac{1}{(n-7)!\,8!} + \cdots,$$

$$(7)\quad \frac{1}{(n-2)!\,3!} + \frac{1}{(n-6)!\,7!} + \frac{1}{(n-10)!\,11!} + \cdots,$$

$$(8)\quad \frac{1}{(n-1)!\,2!} + \frac{1}{(n-5)!\,6!} + \frac{1}{(n-9)!\,10!} + \cdots.$$

2984. Proposed by A. L. CANDY, University of Nebraska.

Find the number of numbers of n digits each that can be written with n consecutive digits, allowing all possible repetitions, such that the sum of the digits in each number is a multiple of n.

2985. Proposed by A. L. CANDY, University of Nebraska.

Find the number of combinations of n digits each that can be made with the first n consecutive digits, allowing repetitions, and such that the sum of the digits in each combination is a multiple of n.

SOLUTIONS.

2890 [1921, 184]. Proposed by B. F. FINKEL, Drury College.

Having given a triangle whose base is $2c$ and (a) the sum of whose other two sides is $2a$, (b) the difference of whose other two sides is $2a$, determine the envelope of the perpendicular bisectors of the variable sides.

I. SOLUTION BY THE PROPOSER.

(a) Letting the vertices of the triangle be $(-c, 0)$, $(c, 0)$ and (α, β), we have

$$\alpha^2/a^2 + \beta^2/b^2 = 1, \tag{1}$$

where $b^2 = a^2 - c^2$.

The equation of the perpendicular bisector of the line connecting $(c, 0)$ and (α, β) may be written[1]

$$(\alpha - x)^2 + (\beta - y)^2 = (x - c)^2 + y^2. \tag{2}$$

Taking α as a variable parameter and differentiating with respect to α, we have $d\beta/d\alpha = -b^2\alpha/a^2\beta = -(\alpha - x)/(\beta - y)$, and our problem is to eliminate α and β.

The elimination may be carried out as follows:

$$\beta = b^2\alpha y/[b^2\alpha - a^2(\alpha - x)],$$

and if we write e for c/a, we have

$$\beta = \frac{(1 - e^2)\alpha y}{x - e^2\alpha} \quad \text{and} \quad \beta - y = \frac{(\alpha - x)y}{x - e^2\alpha},$$

and equations (1) and (2) become

$$a^2 - \alpha^2 = \frac{(1 - e^2)\alpha^2 y^2}{(x - e^2\alpha)^2}, \tag{3}$$

and

$$(x - ae)^2 + y^2 - (\alpha - x)^2 = \frac{(\alpha - x)^2 y^2}{(x - e^2\alpha)^2}; \tag{4}$$

whence

$$\frac{(x - ae)^2 + y^2 - (\alpha - x)^2}{a^2 - \alpha^2} = \frac{(\alpha - x)^2}{(1 - e^2)\alpha^2},$$

and therefore

$$(a^2 - e^2\alpha^2)(\alpha - x)^2 = (1 - e^2)[(x - ae)^2 + y^2]\alpha^2. \tag{5}$$

Now equation (3) is the same as

$$(a^2 - \alpha^2)(e^2\alpha - x)^2 = (1 - e^2)\alpha^2 y^2, \tag{6}$$

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[1] It should be noticed that this solution, as well as the next, gives only one part of the locus, the two parts being symmetrical with respect to the y-axis—EDITORS.